Gait Sequence generation of a Hybrid Wheeled-Legged Robot for negotiating discontinuous terrain

Sartaj Singh, K Madhava Krishna

Abstract—In this paper we develop an algorithm to generate gait sequences to negotiate a discontinuous terrain for a hybrid 4-wheeled legged robot. The gait sequence comprises two main steps – normal force redistribution and hybrid position-force control. The robot climbs the discontinuity one leg at a time. This requires that the entire load of the robot is taken up by the other three legs so that the leg climbing the discontinuity is free. For this purpose a load redistribution methodology is used which makes the center of gravity of chassis coincide with the desired center of pressure (CoP). Subsequently the free leg moves in hybrid position and force control to climb the discontinuity. Force sensing ensures constant contact with the terrain and detection of stand and end of the discontinuity without using any perception sensor. The methodology is validated using multi-body dynamic simulation.

Index Terms—force sensing, force control, legged locomotion, motion control, robot kinematics, wheeled robot.

I. INTRODUCTION

Mobility of wheeled robots traversing on terrain having varying geometrical parameters is the focus of our research. Mobility enhancement of wheeled robots on rough terrain has been accomplished in the past by using passive suspension mechanisms, [1]-[4]. Examples of vehicle systems with passive suspension mechanisms are the Shrimp robots [2] and Rocky rovers [1]. Since these robots had no active reconfiguration mechanism the research was focused on developing traction control algorithms for improving the vehicle stability and power efficiency[5]-[6]. The main disadvantage of passive suspension systems is that the mobility and the stability of these systems under varying terrain conditions cannot be guaranteed. To enhance the performance of such systems a class of robots with actively articulated chassis called the Wheeled and Actively Articulated Vehicles (WAAVs) have been developed. This terminology was first used by Srinivasan et al. [7]. Another class of suspension vehicles are Hybrid Wheel Legged Vehicles (HWLV) such as Hylos [8], PAWS [9] and MHT [10]. Locomotion of Hylos was achieved by a posture control algorithm that uses the velocity model (which maps the joint velocities to the velocity of the main body) to set the velocities at the various joints based on the posture error which in turn maximizes stability and traction of the robot. MHT [10] makes use of velocity model developed in Hylos for inverse kinematics of the vehicle and incorporates optimization technique to minimize joint torques. However the control methodology is limited to rough terrain with no discontinuous features. PAWS [9] can climb steps using optimization techniques to determine the hip joint angles for maximizing height and minimizing traction. PAWS does not use the leg articulation during climbing but depends on wheel traction to climb. Additionally the robot derives the heighth using the motor encoders and IMU. In this paper we present a model of a quadruped HWLV with additional force sensor in each leg. This enables measurement of forces at the wheel-ground contact points. The force values are used to detect the start and end of discontinuity in terrain such as a step. A gait sequence algorithm is presented enabling the climbing of the discontinuity one leg at a time. The algorithm generates gait sequence to maximize the static stability margin of the vehicle while one of the legs climbs the discontinuity using force control along one direction and position control in perpendicular direction. The margin stability used is as defined by Papadopoulos and Rey [11]. Validity of the algorithm has been extensively tested via simulations using MSC VisualNastran and MATLAB/SIMULINK interface. The ability of the vehicle to negotiate discontinuities more than twice the wheel diameter without any information of the discontinuity geometric parameters vindicates the efficacy of the proposed method.

II. NOVELTIES

A key advantage while crafting a gait sequence for a HWLV than a legged robot is that all the legs are in contact with the terrain. This allows for a sequence where both the non-climbing leg (a leg that would not be climbing the discontinuity) as well as the vehicle main body/chassis can be actuated to attain a posture that maximizes the stability margin. Such an option does not exist with the legged robots since any movement of the non-climbing leg immediately affects the vehicle stability. The proposed method depicts how this advantage can be harnessed for each leg that negotiates the discontinuity.

The essential novelties of the paper are argued as follows. Firstly it is one of the few papers that propose a gait sequence to achieve a posture configuration for a desired load distribution amongst the HWLV class of vehicles. Most of the previous methods have focused on achieving a set of joint velocities for a desired velocity of the vehicle main body. These methods do not demonstrate moving over discontinuities while they do show results over uneven terrain. Maintaining a particular configuration of the vehicle main body is the main theme than reconfiguration for a desired force distribution. Secondly the current method uses a simple but elegant combination of linear and torsional springs to maintain terrain contact and detect discontinuities.
III. SYSTEM DESCRIPTION

Fig. 1 shows the mechanical structure of the proposed model of the vehicle. The system has been designed as a hybrid wheeled-legged robot. It consists of four leg-wheel chains connected to the chassis. Each leg has three degrees of freedom consisting of 2-DoF hip joint and a knee joint. The leg-tip has a 2-DoF force sensor comprising a revolute joint loaded with a torsional spring of known stiffness and a prismatic joint loaded with a linear spring of known stiffness. The force acting at the tip of the force sensor is resolved into two components, one parallel and one perpendicular to the prismatic joint. The parallel component is sensed by the linear spring while the perpendicular component is measured by the torsional spring. The tip of the force sensor (called ankle) is fitted with an actuated wheel. Next we derive the forward kinematics of each leg.

A. Leg chain Forward Kinematics

Fig. 2 shows the kinematics of the single leg with the corresponding frame assignments at various joints.

\{W\} is the world frame at the center of the chassis with the Z-axis aligned to gravity. \{P\} is the platform frame with origin coincident with \{W\}. Yaw angle between \{W\} and \{P\} is always zero. \{0\} is the frame attached to fixed end of the hip joint and \{1\}, \{2\} and \{3\} are the frames to each of the rotary joints. Frame \{4\} is attached to the ankle joint where the force sensor is placed and will be explained in detail later.

The DH parameters of this leg are shown in Table I.

![Fig. 1. Mechanical Structure of the HWLV](image1)

![Fig. 2. Kinematics of the single leg chain](image2)

<table>
<thead>
<tr>
<th>i</th>
<th>$a_{i-1}$</th>
<th>$\alpha_{i-1}$</th>
<th>$\theta_i$</th>
<th>$d_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$\theta_1$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$\pi/2$</td>
<td>0</td>
<td>$\theta_2$</td>
<td>$L_1$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$L_2$</td>
<td>$\theta_3$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$L_3$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$L_1 = \pm 0.04 m$, +ve for leg1, leg3 and –ve for leg2 and leg4
$L_2 = 0.125 m$
$L_3 = 0.175 m$ (including force sensor length)

Based on the DH parameters transformation matrix from frame \{0\} to frame \{4\} of the leg can be calculated and represented by \(T\). Transformation from frame \{0\} of leg to frame \{P\} at center of the chassis is given by (1)

\[
{p}_0^T = \begin{bmatrix}
0 & 0 & 1 & L_{x0} \\
0 & 1 & 0 & L_{y0} \\
-1 & 0 & 1 & L_{z0} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(1)

Where,

$L_{x0} = \pm 0.06 m$
+ve for leg1 and leg2 and –ve for leg3 and leg4
$L_{y0} = \pm 0.115 m$,
+ve for leg2 and leg4 and –ve for leg1 and leg3
$L_{z0} = -0.02 m$

are the offsets of frame \{0\} to frame \{P\}. The transformation matrix from frame \{P\} to world frame \{W\} comprise the platform roll (a) and pitch (B) given by.

\[
{w}_R^P = \begin{bmatrix}
1 & 0 & 0 & \cos\beta & 0 & \sin\beta \\
0 & \cos\alpha & -\sin\alpha & 0 & 1 & 0 \\
0 & \sin\alpha & \cos\alpha & 0 & -\sin\beta & 0 & \cos\beta
\end{bmatrix}
\]

(2)

The forward kinematics of the leg tip pose in the world frame is given by

\[
P_{leg} = {w}_R^P {p}_0^T \hat{T}
\]

(3)

Total mass of the chassis including the legs is 7 Kg resulting in a weight of 68.6 N. Mass of each wheel is 1 Kg.

B. Leg Jacobian

If Jacobian of the leg assembly in the frame \{0\} if given by \(\hat{J}\) then jacobian of leg assembly in the world frame \{W\}

\[
\hat{J}_W = \hat{J}_0^W \hat{J}
\]
is given by (4).

\[
\begin{align*}
\text{w}J & = \frac{\text{w}}{\rho} \overrightarrow{p_{\text{rup}}} \text{R} \theta \text{J} \\
\end{align*}
\]  (4)

The Cartesian velocity of leg ankle therefore given by

\[
\begin{bmatrix}
v_x \\
v_y \\
v_z
\end{bmatrix} = \text{w}[\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3]
\]  (5)

Where,

\(\dot{\theta}_1, \quad \dot{\theta}_2, \quad \dot{\theta}_3\) are angular velocities of the joints of leg.

C. Leg Force Sensor

The leg force sensor is shown in Fig. 3.

In the ankle frame \(\{4\}\) we have

\[
\begin{align*}
4F_x & = K_{st}(L - L_0) \\
4F_y & = -K_{st}(\theta - \theta_0)/(r + L) \\
4F_z & = 0 \\
4F_{\text{ankle}} & = \begin{bmatrix} 4F_x & 4F_y & 4F_z \end{bmatrix}
\end{align*}
\]

Where,

\(K_{st}\) : Linear spring constant = 5000 \(N/m\)  \\
\(L\) : Linear spring length in meters  \\
\(L_0\) : Linear spring normal length in meters  \\
\(\theta\) : Torsional spring angle in radians  \\
\(\theta_0\) : Torsional spring normal angle in radians  \\
\(r\) : Fixed length in meters

The force sensor values in the \(\{W\}\) can now be calculated as:

\[
\begin{align*}
wF & = \frac{w}{\rho} r \overrightarrow{p_{\text{rup}}} 4F_{\text{ankle}} \\
\end{align*}
\]

Where, \(\text{R}\) is rotation between frame \(\{i\}\) and \(\{j\}\).

The force given by \(wF\) acts at the ground-wheel contact minus the weight of the wheel. Without loss of generality this force can be considered as indicative of contact force.

IV. CONTROL SYSTEM

A hierarchical control system is implemented for the wheeled-legged robot. The lowest level is the angular position/velocity control of all joints. Next level is the leg tip velocity/position control shown in Fig. 4. Highest level controller is the robot chassis position control.

The joint position controller uses PD control law. Joint velocity control is achieved by integrating velocity and using the position controller.

Leg tip velocity controller uses the leg jacobian to determine the joint angular velocities which are controlled by joint velocity controller. Leg tip position control is achieved by driving the leg velocities based on the error in leg tip position. The wheel angular velocity \(\omega\) is determined using the leg velocity vector \(\overrightarrow{v_{\text{leg}}}\) and contact force vector \(\overrightarrow{F}\) defined in \(\{W\}\). Magnitude of \(\omega\) is proportional to the magnitude of velocity vector while direction of rotation is provided by the cross product of force and velocity vector.

\[
\begin{align*}
||\omega|| & = \frac{||\overrightarrow{v_{\text{leg}}}||}{r_w} \\
\text{Direction}(\omega) & = \text{Direction}(\overrightarrow{F} \times \overrightarrow{v_{\text{leg}}})
\end{align*}
\]  (7)

Where, \(r_w\) is the wheel radius.

There is no mechanism/sensor for determining the absolute value of the chassis. Hence only relative position control is considered. Relative change in position of the chassis can be achieved by giving the negative delta change to the leg position controllers while keeping the wheels locked. Friction between the wheels and ground keeps the contact points fixed resulting in the motion of the chassis in opposite direction.

V. STEP CLIMBING GAIT SEQUENCE

In this section we present the step climbing gait sequence algorithm. The robot starts moving with position of all four legs in nominal state and all wheels rotating with fixed angular velocity. Presence of discontinuity is detected when a leg experiences high horizontal force for more than a predefined period of time. The wheels are stopped and whole weight of the robot is shifted to remaining three legs using force redistribution algorithm. Finally the free leg climbs the step using hybrid position-force control.

A. Force Redistribution Phase

In force redistribution total weight of the robot is distributed amongst three legs called support legs. This requires motion of the robot chassis such that its XY position coincides with XY location of the center of pressure (CoP) formed by three legs. CoP is defined as the point on
the contact plane formed by the contact points of the legs about which the moments due to normal forces are zero.

For given contact points \((x_i, y_i)\) and the normal force \(f_{zi}\) at these points, location of CoP \((x_{pr}, y_{pr})\) is determined using moment balance given by (8) where sum is taken for all legs.

\[
\sum (y_i - y_{pr}) f_{zi} = M_x = 0 \\
\sum (x_i - x_{pr}) f_{zi} = M_y = 0
\]

From the above equation we have

\[
x_{pr} = \frac{\sum f_{zi} x_i}{\sum f_{zi}} \\
y_{pr} = \frac{\sum f_{zi} y_i}{\sum f_{zi}}
\]

The objective is to determine a CoP which results in high margin of stability (MoS) by relocating the contact points. Margin of stability is defined as the minimum of all perpendicular distances of the CoP from sides of the polygon formed by the contact points [11].

The CoP and the MoS with four legs of the robot in contact are depicted in Fig. 5. MoS is minimum of \((P_1, P_2, P_3, P_4)\)

Let us assume that leg1 detects the step. For distributing the robot weight amongst remaining three legs the desired normal force for the four legs would be 0 on leg1 and W/3, W/3, W/3 on the other legs, where ‘W’ is the total weight of the robot. Next the XY location of the center of pressure for desired forces is calculated using (9). For equal distribution of weight amongst three legs the CoP \((x_{pr}, y_{pr})\) is the centroid of the triangle formed by the three legs. Fig. 6 shows the location of the CoP for nominal position of the legs and the movement of chassis, \(F_{pr}\), required to achieve this CoP.

In nominal state of the legs location is :

\[
(x_2, y_2) = (0.15576, 0.155) \\
(x_3, y_3) = (-0.15576, -0.155) \\
(x_4, y_4) = (-0.15576, 0.155) \\
D_{pr} = 0.07324 m \\
MoS = 0.07324 m
\]

As can be seen this requires a movement of chassis center by 0.07324 m. It was found that movement is so large that the legs reach the workspace singularity before the desired position is achieved as shown in Fig. 7.

![Fig. 6. Center of Pressure (CoP) with leg2, leg3 and leg4 in contact and in nominal position](image)

In order to minimize the chassis motion we first relocate the contact points of the support legs. Additionally, the new contact points should result in CoP having high margin of stability (MoS). It is preferred to relocate the ‘x’ coordinates of the legs so that leg motion is required only along the sagittal plane of the robot. The wheels are not provided with steering and motion with pure rolling is possible only in the sagittal plane. The problem is therefore posed as an optimization problem which minimizes the chassis motion and maximizes the stability margin of the vehicle.

The objective function for minimization is given by (10).

\[
f = D_{pr} - \min(P_3, P_2, P_3)
\]

Where, \(D_{pr}\) is the distance between chassis center and CoP \(P_i\) is the perpendicular distance from CoP to the line \(L_i\) of the triangle formed by the support legs as shown in Fig. 6.

Equality constraints of the problem are given by (9) and the inequality constraints are given by lower and upper bounds on the ‘x’ coordinates of the legs defined by the kinematic boundary singularities.

Output of the optimization results in a solution where the leg2 and leg4 contact points remain unchanged while the leg3 ‘x’ value is relocated to the middle. The new leg configuration is shown in Fig. 8.
In the optimized state
\[(x_3, y_3) = (0, 0.155)\]
\[D_{pr} = 0.05166 \text{ m}\]
\[MoS = 0.09278 \text{ m}\]
The chassis requires 1.4 times lesser movement and the MoS is improved 1.3 times as compared to nominal state.

The leg3 is moved to new \((x_3, y_3)\) location using leg control scheme described in section IV to achieve the final configuration shown in Fig. 8. Next the chassis center is moved to the desired CoP \([x_P, y_P]\) using body position control scheme described in section IV. In this configuration the robot has complete weight supported by three legs with maximum stability margin. Here effect of mass of leg due to relocation on CoP is ignored assuming that leg mass is much less as compared to the mass of chassis.

**B. Leg Climbing Phase**

Once the force redistribution phase is complete the leg which detected the beginning of discontinuity is free to climb. Climbing is achieved by hybrid position-force control. The leg is moved with force control along the horizontal direction and a constant velocity along the vertical direction. For force control the leg horizontal velocity is driven proportional to the error in force. The desired velocity for leg ankle is given by (11).

\[
\begin{align*}
    v_x &= k_p(f_{desired} - f_x) \\
    v_y &= 0 \\
    v_z &= v_{zdesired}
\end{align*}
\]

Where,
\[f_{desired} :\] is the desired horizontal force. The value of the desired force is chosen so that it is less than the total tractive force required for keeping the robot chassis static in global frame.
\[f_x :\] is the horizontal force component measured by force sensor given by (6)
\[v_z :\] is the desired rate of climb

End of the step is detected when horizontal force cannot be maintained. The leg whose contact point is relocated in normal force redistribution phase is moved back to the nominal state. The chassis moved back to nominal state. The robot then moves forward till the second leg experiences the horizontal force and the sequence is repeated. Similar process is followed for the rear legs.

The algorithm adopted for step climbing is given below:

: **Start**

Do{ Move the robot with constant wheel velocity; 
} While no high horizontal force on any leg;

Stop wheels;

Calculate the leg positions of remaining three legs and motion of the chassis for desired CoP using optimization;

Move the legs to the new position using leg position control;

Move the chassis for desired CoP;

Do{ Move the free leg with force control in horizontal direction and velocity control in vertical direction; 
} While no horizontal force can be maintained;

Goto Start

**VI. SIMULATION**

We performed all simulations using MSC VisualNastran with a MATLAB/SIMULINK interface. In Fig. 9 the snapshots of the gait sequence for leg1 climbing the step are given.

Fig. 10 shows the forces measured by the force sensors on leg1. Beginning of the step is detected when horizontal force \(F_x\) is more than 5N in negative direction. Force redistribution algorithm is executed next. The normal force \(F_z\) reduces from 17N to -9.8N (negative due to weight of the wheel). Next the leg1 climbs the step maintaining a contact force of -3N. End of step is detected when contact can no more be maintained.

![Fig. 9. Snapshots of the simulation of leg1 climbing a step](image-url)
Fig. 10. Horizontal and vertical forces measured by force sensor on leg1 during step climbing sequence

Fig. 11. Horizontal and vertical forces measured by force sensor on leg2, leg3 and leg4 during step climbing by leg1

VII. CONCLUSION

The step climbing gait sequence proposed for a HLWV in this paper has been validated through simulation. It is shown that using normal force redistribution followed by force and position based control the HLWV is able to climb the step one leg at a time without affecting the stability of the robot and without the need of any other perception sensors. As future work the methodology developed in this paper will be implemented on the real robot being developed. Effect of mass of the legs will also be taken into consideration during this implementation to ensure stability.

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IX. REFERENCES


